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Description of a Mathematical Model of Deformability for the Process of Drawing Tubes on a Fixed Mandrel

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This article presents a generalized model devised by W.L. Kolmogorow to describe deformability of metal in the process of drawing tubes on a fixed mandrel and different factors affecting the utilization of reserve of plasticity. It demonstrates that it is possible to model this process by means of a simple mechanical test.

Keywords deformability, drawing, drawing tubes, fixed mandrel, metal forming

1. Introduction

In recent years, rapid development of computer techniques and the application of the theory of plasticity has made it possible to use a more complex approach to problems of deformability and plasticity of metals. In the existing concepts (Ref 1-3), attempts were made to solve the problem of allowable strains either by taking into account the history of deformation, i.e., relations between stresses and strains during plastic deformation of metals, or by drawing some interesting conclusions relating to fracture of metals from the history of deformation (Ref 4, 5). The latter constitutes a novel approach relative to an earlier method based on the assumption that a certain physical value, e.g., the maximum tensile stress, is responsible for fracture of metals. Then the critical value of this parameter was determined from one of the tests and comparison was made with the theoretical value derived for a given technological process. Correctness of this formula was proof that the assumption made was right.

A theory developed by Kolmogorow (Ref 1, 2), using the factor of utilization of reserve of plasticity, permits one to choose the best technology (in terms of deformability) for a given plastic working process. However, Kolmogorow, deriving formulas for particular processes, introduced not only qualitative simplifications (e.g., he made an assumption that the material is homogeneous) but also quantitative simplifications (e.g., he omitted some terms of a formula or made calculations for constant conditions characterizing a process, for example at a constant coefficient of friction). These simplifications result in a lesser accuracy of formulas, and they do not permit analysis of the logical correctness of a formula.

Cockcroft and Latham (Ref 3) assumed that the amount of work made by the maximum tensile stress until the moment of fracture is a constant value and independent of the type of test. On the basis of this assumption the allowable strains can be determined by comparing the calculated theoretical amount of work for a given technological process with that obtained experimentally during a test. To date, however, a confirmation that this assumption is right is lacking. For the rest, experimental determination of work from any mechanical test is troublesome and involves a great error.

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Nomenclature	
ψ	Total coefficient of utilization of plasticity reserve in the process of drawing tubes on a fixed mandrel
ψ_r	Coefficient of utilization of plasticity reserve in the die reduction zone
ψ_0	Coefficient of utilization of plasticity reserve in the die sinking zone
t	Time
t_1	Time at which the fracture of material occurred
$\dot{\gamma}_i$	Intensity of effective strain rate
g_i	Degree of effective strain
g_{iz}	Effective strain at fracture
σ_r, σ_θ	Radial and circumferential stresses
$k = \dfrac{\sigma_m}{\tau_i}$	Stress state factor
σ_m	Mean stress
τ_i	Intensity of shearing stresses
$B(t)$	A function describing a nonmonotonic progress of deformation
$\dot{\varepsilon}_r$, $\dot{\varepsilon}_\theta$	Radial and circumferential strain rates
α	Angle of die reduction zone
f	Coefficient of friction
d_0	Ingoing outside diameter of tube
d_1	Outgoing outside diameter of tube
s_0	Initial (ingoing) tube wall thickness
s_1	Wall thickness at the beginning of the die sinking zone
s_2	Wall thickness of finished tube on leaving the sinking zone
v_1	Flow velocity (along x axis) of tube on leaving the die
F_x	Cross-sectional area of tube at any cross section (defined by variable x)
F_0	Cross-sectional area of ingoing tube
F_1	Cross-sectional area of finished tube
λ	Coefficient of elongation
e_0, e_1, e_2	Coefficients occurring in approximation of curve of deformability, usually determined by the least-squares method
R_e	Yield stress in uniaxial tension test

The author decided to generalize and to complete the model of deformability proposed by Kolmogorow (Ref 2) for the process of drawing tubes on a fixed mandrel, and then to program this model in Algol 1204 language. With the programmed model, effects of various factors on the coefficient of utilization of the reserve of plasticity, and thereby the deformability of metal in this process, have been investigated. The practical interest in the availability of such a program consists in making very rapid calculations (requiring several hours) for proposed drawing techniques and in making an optimum choice thereof (Ref 6, 7).

2. Description of the Applied Model

A mathematical description of stress and strain states in the process of drawing tubes on a fixed mandrel is a difficult task, because in this process there occur two deformation zones (Fig. 1), with different schemes of stress and strain. This does not permit one to make use of continuous functions to describe this process, and it complicates calculations. For the mathematical (quantitative) determination of deformability of metal, the factor of utilization of plasticity reserve has been used. This factor is defined by (Ref 2):

$$\psi = \int_0^{t_1} B(t)\,\frac{\dot{\gamma}_i(t)}{g_{iz}[k(t)]}\,dt \qquad \text{(Eq 1)}$$

On the basis of our own investigation (Ref 6, 7), assumption was made that the curve of deformability can be approximated by a parabola (often simplified to a straight line), in the following form:

$$g_{iz} = e_0 + e_1 k + e_2 k^2 \qquad \text{(Eq 2)}$$

From analysis of Eq 1 it follows that to calculate the factor of utilization of reserve of plasticity, one must know the intensity of strain rate and the state of stress factor for both deformation zones. Knowing these magnitudes, the factor of utilization of plasticity reserve has been calculated from:

$$\psi = \psi_r + \psi_0 \qquad \text{(Eq 3)}$$

In deriving the corresponding equations, the following assumptions have been made:

- The metal does not strain harden during deformation. This assumption was made on account of the complete lack of published data relating to the influence of strain hardening on the curve of deformability. This effect can eventually be taken into account in a further development of the model, based on the concept described in Ref 8.
- The deformation progress has a monotonic character, which means that $B(t) = 1$. A non-monotonic character of deformation in the process of drawing tubes on a fixed mandrel is caused by additional shearing stresses and strains occurring during deformation of metal.
- The metal is homogeneous and there is lack of anisotropy.
- In the die reduction zone, $\sigma_r = 0$ (Ref 2).
- The outside tube diameter at the beginning of the sinking zone is equal to the outside diameter of the finished tube (Ref 2).

How do these assumptions affect the accuracy of calculations? It is well known that properties of materials change differently depending on the degree of strain hardening. Therefore an error caused by neglecting the phenomenon of strain hardening of metal during the deformation process will be a function of the kind of metal. As mentioned above, publications on the influence of strain hardening on the curve of deformability are lacking. However, it is known (Ref 9) that as the cold work rises the tensile strength is increased and diminishes the elongation and impact strength of metals. It follows that making this assumption will result in a determination of deformability with some deficiency.

As regards the evaluation of an error caused by assuming a monotonic character of deformation progress, this problem was solved only several years ago and that is why more accu-

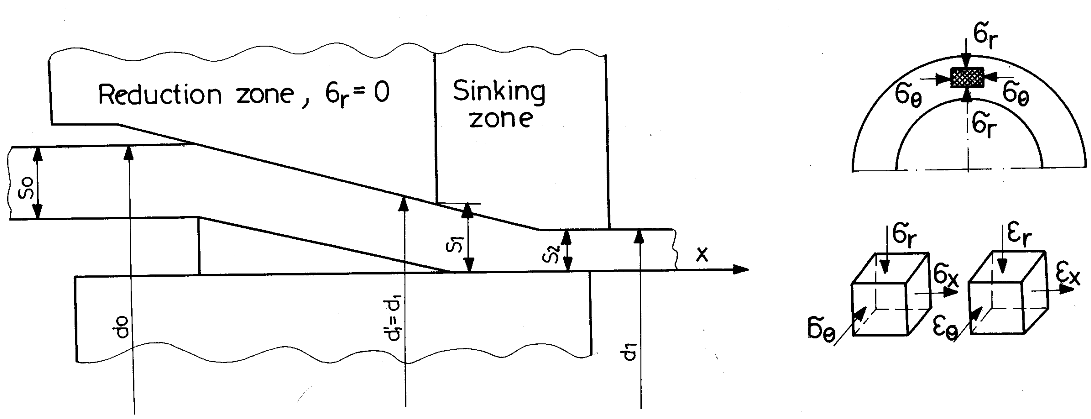

Fig. 1 Schematic representation of drawing tubes on a fixed mandrel

rate data on this subject are lacking, too. Among the interesting publications devoted to this problem is the paper by Blazynski and Cole (Ref 10), where the additional work caused by additional shearing strains is given to be about 8 p.c. Renne (Ref 11) has calculated the intensity of strain in the process of drawing tubes on a floating mandrel for the reduction zone angle 18° and tube wall reduction 50 p.c. The intensity of strain calculated from the formula for homogeneous plane deformation was 1.18 and amounted to 1.52 when calculated from grid distortions and taking into account the history of deformation. On the basis of the above publications it can be said that assumption of a monotonic nature of deformation progress in the process of drawing tubes on a fixed mandrel results in an error of 10 to 20 p.c. in minus in determining components of strain and stress states.

Assumption that the material is homogeneous and isotropic readily suits single-phase high-purity materials as well as annealed steels. However, as regards the hot-rolled metals used for drawing without prior annealing, the nonhomogeneity of such materials has to be taken into consideration. The error resulting from this assumption depends on the method of determining the curve of deformability. The metal properties, as determined on specimens cut from areas revealing the worst properties, will minimize the error to a reasonable extent.

The assumption that $\sigma_r = 0$ in the reduction zone, with reference to thin-walled tubes (Ref 2), will result in a considerable error in making calculations for determining the coefficient of utilization of plasticity reserve for thick-walled tubes. The assumption that the outside diameter of tube at the beginning of the sinking zone is equal to that of the finished tube will not cause a greater error because the change of tube diameter in this zone is quite small (Ref 2).

The verification of the model as made for tubes of K18 steel revealed that calculation results are, in general, in good accordance with reality. This permits us to say that the above simplifications involve errors that cancel each other. A fault of the model is an erroneous calculation of the influence of friction.

To calculate the coefficient of utilization of plasticity reserve in the process of drawing tubes on a fixed mandrel in accordance with Eq 1, parameters characterizing the stress and strain states in both the deformation zones have to be determined. The intensity of strain rate (describing the strain state) in the reduction zone has been calculated according to Ref 2 and 12:

$$\dot{\varepsilon}_r = \frac{1 - 3(1 - 2s_0/d_0)\,(d/d_0)^{2(1 + f\,\mathrm{ctg}\,\alpha)}}{1 + 3(1 - 2s_0/d_0)^2\,(d/d_0)^{2(1 + f\,\mathrm{ctg}\,\alpha)}}\,\dot{\varepsilon}_\theta \quad \text{(Eq 4)}$$

$$\dot{\varepsilon}_\theta = \frac{2v_x\,\mathrm{tg}\alpha}{d} \quad \text{(Eq 5)}$$

$$\dot{\gamma}_i = 2\sqrt{\dot{\varepsilon}_r^2 + \dot{\varepsilon}_r\dot{\varepsilon}_\theta + \dot{\varepsilon}_\theta^2} \quad \text{(Eq 6)}$$

On substituting Eq 4 into Eq 6 one obtains finally:

$$\dot{\gamma}_i = 2\dot{\varepsilon}_\theta\,\sqrt{u^2 + u + 1} \quad \text{(Eq 7)}$$

where the variable u is determined as follows:

$$u = \frac{1 - 3(1 - 2s_0/d_0)\,(d/d_0)^{2(1 + f\,\mathrm{ctg}\,\alpha)}}{1 + 3(1 - 2s_0/d_0)^2\,(d/d_0)^{2(1 + f\,\mathrm{ctg}\,\alpha)}} \quad \text{(Eq 8)}$$

The stress state factor for this zone is calculated according to (Ref 2):

$$k = -2\,\frac{\dot{\varepsilon}_r}{\dot{\gamma}_i} + \frac{\sigma_r}{\sigma_m} \quad \text{(Eq 9)}$$

Because $\sigma_r = 0$, hence finally:

$$k = -2\,\frac{\dot{\varepsilon}_r}{\dot{\gamma}_i} = -\frac{u}{\sqrt{u^2 + u + 1}} \quad \text{(Eq 10)}$$

The coefficient of utilization of plasticity reserve in the reduction zone is calculated by substituting Eq 7 and 10 into Eq 1.

After transformations one obtains:

$$\psi_r = \int_{d_1/d_0}^{1} 2\,\frac{\sqrt{u^2 + u + 1}}{e_0 + e_1\,(-u/\sqrt{u^2 + u + 1}) + e_2\,[u^2/(u^2 + u + 1)]}\,\frac{dz}{z} \quad \text{(Eq 11)}$$

where $z = d/d_0$.

In the reduction zone occurs a thickening of tube wall due to deformation. This thickening has been calculated from (Ref 2, 12):

$$\ln\frac{s_1}{s_0} = \frac{\ln\left(\frac{d_0}{d_1}\right)^{2(1 + f\,\mathrm{ctg}\,\alpha)} - \left(2 - \frac{2s_0}{d_0}\right)\ln\frac{3\left(1 - \frac{2s_0}{d_0}\right)^2 + \left(\frac{d_0}{d_1}\right)^{2(1 + f\,\mathrm{ctg}\,\alpha)}}{3(1 - 2s_0/d_0)^2 + 1}}{2(1 + f\,\mathrm{ctg}\,\alpha)\left(1 - \frac{2s_0}{d_0}\right)} \quad \text{(Eq 12)}$$

The intensity of strain rate in the sinking zone has been calculated, assuming according to Ref 2 that:

$$\dot{\gamma}_i = 2\dot{\varepsilon}_x = 2\frac{\partial v_x}{\partial_x} \quad \text{(Eq 13)}$$

The strain rate v_x is calculated on the basis of the constant-flow principle, assuming the amount of material flowing through any section in a time unit to be constant (Ref 6, 7):

$$F_x v_x = \text{constant} \quad \text{(Eq 14)}$$

Hence one can write:

$$\frac{\pi}{4} v_1 d_1^2 - v_1 \frac{\pi}{4}(d_1 - 2s_2)^2 = \frac{\pi}{4} v_x(d_1 - 2s_2 + 2s)^2 - \frac{\pi}{4} v_x(d_1 - 2s_2)^2 \quad \text{(Eq 15)}$$

After transformations one obtains:

$$v_x = \frac{4v_1 d_1 s_2 - 4v_1 s_2^2}{4s^2 + 4d_1 s - 8s_2 s} \quad \text{(Eq 16)}$$

With known v_x, the term $\dot{\varepsilon}_x$ is to be calculated:

$$\dot{\varepsilon}_x = \frac{dv_x}{d_x} = \frac{dv_x}{ds}\frac{ds}{dx} = \frac{(4d_1 v_1 s_2 - 4v_1 s_2)\,(8s + 4d_1 - 8s_2)}{(4s^2 + 4d_1 s - 8s_2 s)}\frac{ds}{dx} \quad \text{(Eq 17)}$$

The next step in deriving an algorithm consists of calculating:

$$dt = \frac{dx}{v_x} \quad \text{(Eq 18)}$$

$$\dot{\gamma}_i dt = \frac{4s_2 - 2d_1 - 4s}{s^2 + d_1 s - 2s_2 s} ds \quad \text{(Eq 19)}$$

The stress state factor for the sinking zone is calculated as follows:

$$k = \frac{\sqrt{3}}{3R_e}(\sigma_x - 2\sigma_r) \quad \text{(Eq 20)}$$

The condition of plasticity in this zone is defined by:

$$\sigma_r = \frac{2}{\sqrt{3}} R_e - \sigma_x \quad \text{(Eq 21)}$$

By substituting Eq 21 into Eq 20 one obtains:

$$k = \sqrt{3}\,\frac{\sigma_x}{R_e} - \frac{4}{3} \quad \text{(Eq 22)}$$

On substituting equation defining σ_x (Ref 13) into Eq 22 and making necessary transformations one obtains:

$$k = \frac{2\varepsilon_1}{\varepsilon_1 - 1} - \frac{2\varepsilon_1}{\varepsilon_1 - 1}\left(\frac{s_1}{s}\right)^{(\varepsilon_1 - 1)} + 2\,\frac{\varepsilon}{\varepsilon - 1}\left(\frac{s_1}{s}\right)^{(\varepsilon_1 - 1)} - \frac{2\varepsilon}{\varepsilon - 1}\left(\frac{s_1}{s}\right)^{(\varepsilon_1 - 1)}\left(\frac{d_1}{d_0}\right)^{(\varepsilon - 1)} - \frac{4}{3} \quad \text{(Eq 23)}$$

where:

$$\varepsilon_1 = \frac{f + \text{tg}\,\alpha}{(1 - f\,\text{tg}\,\alpha)\text{tg}\,\alpha} + \left(1 - 2\,\frac{s_2}{d_1}\right)\frac{f}{\text{tg}\,\alpha}$$

$$\varepsilon = \frac{f + \text{tg}\,\alpha}{(1 - f\,\text{tg}\,\alpha)\,\text{tg}\,\alpha}$$

The coefficient of utilization of plasticity reserve in the sinking zone is determined from:

$$\psi_0 = \int_{s_1}^{s_2} \frac{\dot{\gamma}_i\,dt}{e_0 + e_1 k + e_2 k^2} \quad \text{(Eq 24)}$$

The total coefficient of utilization of plasticity reserve in the process of drawing tubes on a fixed mandrel is calculated by substituting Eq 11 and 24 into Eq 3.

3. Description of the Program

The model presented above is very complicated and hence there is only one real way to make the calculations, i.e., by computer. All calculations were made on a digital computer of the ODRA 1204 type. The program was prepared in Algol language (Algol 1204 in realization). The occurrence of twofold integration in calculations necessitated the application of a system composed of two programs (output of one program is input of the other). Integration was made by using a procedure from a library of programs. Both the integrals have been calculated with a relative error of 10^{-7}.

Input and output data were floating-point numbers. The input data were the four values representing the outside diameters of tube before and after drawing and the thicknesses of tube wall before and after drawing. The output data were the thickness of tube wall after deformation in the first zone and the three values of coefficient of utilization of plasticity reserve (i.e., value in the first zone, value in the second zone, and the total value of this coefficient) (Ref 6-7).

4. Description of the Data

Calculations were made for several selected cases of drawing thin- and thick-walled tubes. Included were cases of drawing at different coefficients of elongation and different combinations of relative changes of outside diameter and wall thickness (Table 1). Calculations were also made for the drawing techniques given by Blazynski and Cole (Ref 10) (the last six cases from Table 1), with a constant proportion of sink drawing and a variable proportion of the sinking zone.

5. Discussion of the Results

Calculations were made for 16 cases with different coefficients of friction (0.04, 0.08, 0.15, 0.20) and different angles of die reduction cone (6°, 9°, 12°, 16°) by combining these values, each with each. These calculations were made for a steel with high plasticity, $e_0 = 2$. Some results are given in Table 2 and Fig. 2.

In analyzing the values from Table 2 it can be seen that at a constant coefficient of friction, the value of coefficient of utilization of plasticity reserve increases with the increase of angle of the die reduction cone. In contrast, at a constant angle of the die reduction cone, the increase of coefficient of friction is accompanied by a decrease of the ψ value, this being a deficiency of the presented model, because an increase of friction negatively affects the drawing process.

To establish optimum conditions for drawing tubes on a fixed mandrel, technologies with identical coefficients of elongation but revealing different changes in outside diameter and tube wall thickness (cases 6, 11 and 1, 12) were selected. Cases 1 and 12 are characterized by similar values of s_0/s_2 and different values of d_0/d_1. More favorable is case 1, with a smaller change of outside diameter and hence with a smaller change of thickness s_1.

A comparison of respective cases 6, 11 and 1, 12 indicates that in designing a technology of drawing tubes on a fixed mandrel, small reductions of outside diameter and large changes of wall thickness should be taken. Confirmation of this is provided by cases 3 and 15. As can be seen from Table 1, λ_{15} is greater than λ_3; hence, ψ_{15} should be greater than ψ_3. This does not occur, however, because case 15 is characterized by a relatively small change of outside diameter and a large change of wall thickness, or conditions of optimum technology are ful-

Table 1 Data relating to investigated drawing processes

Item	Dimensions of ingoing tube, mm	Dimensions of finished tube, mm	$z = d_0/d_1$	$s = s_0/s_1$	F_0/F_1
1	57 × 4.2	53 × 2.7	0.93	0.64	1.71
2	57 × 4.2	53 × 3.9	0.93	0.93	1.16
3	57 × 4.2	36.6 × 2.7	0.64	0.64	2.42
4	57 × 4.2	36.6 × 3.9	0.64	0.93	1.62
5	57 × 4.2	45.6 × 3.36	0.80	0.80	1.56
6	57 × 4.2	45 × 3.4	0.79	0.81	1.61
7	57 × 1.2	45.6 × 0.96	0.80	0.80	1.56
8	57 × 1.2	53 × 1.12	0.93	0.93	1.17
9	57 × 1.2	36.6 × 1.12	0.64	0.93	1.67
10	55 × 3.66	48.4 × 2.83	0.88	0.77	1.48
11	55.6 × 3.92	48.4 × 2.83	0.87	0.72	1.60
12	56 × 4.21	48.4 × 2.83	0.86	0.67	1.74
13	56.7 × 4.51	48.4 × 2.83	0.85	0.63	1.91
14	57.2 × 4.82	48.4 × 2.83	0.84	0.59	2.06
15	58.2 × 5.30	48.3 × 2.83	0.83	0.53	2.55

Source: Ref 6

Table 2 Calculated coefficients of utilization of plasticity reserve

			f = 0.04						f = 0.20					
			α = 6° (0.1047 rad)			α = 16° (0.2793 rad)			α = 6° (0.1047 rad)			α = 16° (0.2793 rad)		
Item	Dimensions of ingoing tube, mm	Dimensions of finished tube, mm	ψ_r	ψ_0	ψ	ψ_r	ψ_0	ψ	ψ_r	ψ_0	ψ	ψ_r	ψ_0	ψ
1	57 × 4.2	53 × 2.7	0.068	0.390	0.459	0.069	0.400	0.469	0.068	0.318	0.386	0.068	0.375	0.443
2	57 × 4.2	53 × 3.9	0.068	0.096	0.165	0.069	0.097	0.165	0.068	0.092	0.160	0.068	0.095	0.164
3	57 × 4.2	36.6 × 2.7	0.430	0.501	0.931	0.423	0.524	0.947	0.489	0.386	0.875	0.440	0.476	0.916
4	57 × 4.2	36.6 × 3.9	0.430	0.161	0.591	0.423	0.184	0.608	0.489	0.008	0.497	0.440	0.130	0.570
5	57 × 4.2	45.6 × 3.36	0.210	0.275	0.485	0.209	0.282	0.491	0.218	0.239	0.457	0.210	0.268	0.479
6	57 × 4.2	45 × 3.4	0.222	0.269	0.491	0.222	0.276	0.497	0.232	0.230	0.462	0.223	0.261	0.485
7	57 × 1.2	45.6 × 0.96	0.209	0.264	0.473	0.209	0.269	0.479	0.215	0.231	0.446	0.210	0.257	0.467
8	57 × 1.2	53 × 1.12	0.069	0.087	0.156	0.069	0.088	0.157	0.068	0.084	0.152	0.069	0.087	0.155
9	57 × 1.2	36.6 × 1.12	0.425	0.161	0.586	0.420	0.180	0.601	0.479	0.027	0.506	0.434	0.135	0.569
10	55 × 3.66	48.4 × 2.83	0.120	0.270	0.390	0.120	0.275	0.395	0.121	0.243	0.364	0.120	0.265	0.385
11	55.6 × 3.92	48.4 × 2.83	0.130	0.330	0.460	0.130	0.337	0.467	0.131	0.289	0.420	0.130	0.322	0.452
12	56 × 4.21	48.4 × 2.83	0.137	0.389	0.526	0.137	0.398	0.535	0.138	0.331	0.469	0.137	0.377	0.514
13	56.7 × 4.51	48.4 × 2.83	0.148	0.447	0.596	0.148	0.459	0.608	0.151	0.369	0.519	0.149	0.420	0.579
14	57.2 × 4.82	48.4 × 2.83	0.157	0.501	0.657	0.157	0.516	0.673	0.159	0.401	0.560	0.157	0.479	0.636
15	58.2 × 5.3	48.3 × 2.83	0.175	0.578	0.753	0.175	0.599	0.774	0.180	0.444	0.623	0.175	0.548	0.723

Note: These calculations were made for a steel with $e_0 = 2$, $e_1 = -0.3$, and $e_2 = 0$. Source: Ref 6 filled. Similar in this respect are cases 4 and 13, which for α = 16° and f = 0.04 reveal equal coefficients of utilization of plasticity reserve in spite of different coefficients of elongation.

At identical values of z and s (e.g., cases 2 and 3), ψ_0 is greater than ψ_r, which is conceivable due to thickening of tube wall at the exit from the reduction zone of the die.

A decision was also made to verify how particular coefficients from the equation for a deformability curve affect the coefficient of utilization of plasticity reserve. For this purpose calculations were made for three curves: one parabola and two straight lines. Coefficients occurring in these equations were taken from Ref 2, so they correspond to actual materials. The following equations were used:

$$g_{iz} = 1.4306 - 0.5940k + 0.1176k^2$$
$$g_{iz} = 1.9 - 1.1k$$
$$g_{iz} = 2 - 0.3k$$

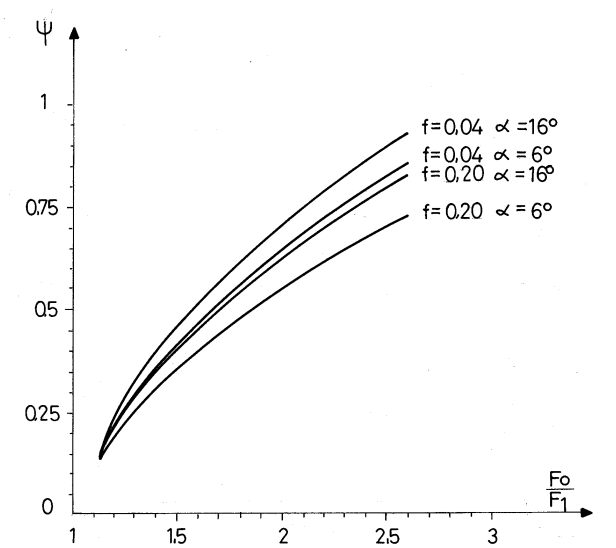

Fig. 2 Influence of coefficient of friction and angle of die reduction zone on coefficient of utilization of plasticity reserve

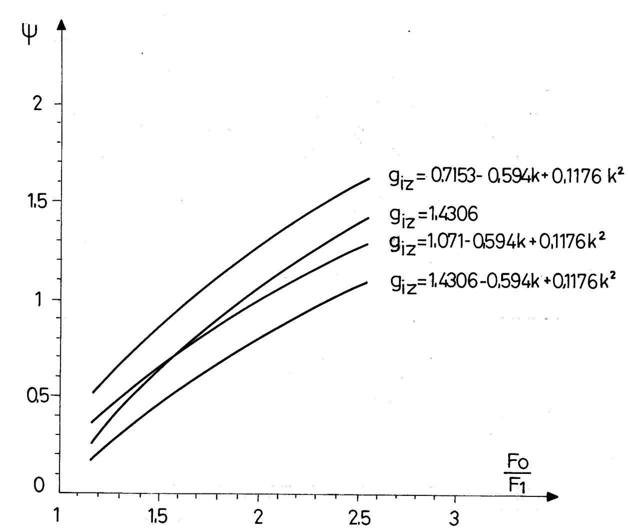

Fig. 3 Effect of coefficients from the equations defining the curve of deformability on the coefficient of utilization of plasticity reserve

All calculations were made for the most frequent case of drawing, where f = 0.08 and α = 12°. Calculations of ψ were made for the above values of coefficients and also for e_1 and e_0 being equal to zero. Further, e_0 was changed by 25 and 50 p.c., respectively. Results of these calculations are given in Table 3 and Fig. 3.

When considering data from Table 3 it may be noticed that a change of e_1 and e_2 by 100 p.c. (i.e., zeroing of these values) results in a small change, about 10 p.c., of the coefficient of utilization of plasticity reserve for the case of drawing defined by a small value of z and a high or medium value of s. For this type of drawing process, the value that decides the deformability is e_0. This follows probably from the zero value of the stress state factor for these cases of drawing tubes on a fixed mandrel. Because the value of e_0 is determined directly from the torsional test on cylindrical specimens, this may mean that such drawing technologies can be modeled by means of the torsion test. For the remaining cases of drawing, the error involved by zeroing of e_1 and e_2 (or only of e_1, for a straight line) is very great. It was a reason that investigation was made to establish which test is the best to determine boundary deformability in cold metal forming processes (Ref 14-18).

We also studied the effect of changing e_0 (at constant values of e_1 and e_2) on the coefficient of ψ. It may be seen from Table 3 that the change of e_0 by 25 p.c. causes, for a given grade of steel, an approximately constant and great error. At the change of e_0 by 50 p.c. the error increases still further (attaining the value of 100 p.c.) and also increases the extent of the error between particular cases of drawing.

When studying the effect of e_0, e_1, and e_2 on the coefficient ψ, attention was also paid to the influence of the steel grade. The smallest errors occurred in the case of a material with e_0 = 2, e_1 = –0.3, and e_2 = 0. This was probably due to a small value of e_1, deciding the slope of the curve. Of course, modeling of the process for a material with a small slope of the deformability curve will be the easiest and the most accurate. In this instance the amount of the effective strain will be approximately constant for all the stress states, and for modeling a technological process any mechanical test can be used.

Thus the following can be said (Ref 6, 7):

- For certain cases of drawing (defined by a small value of z and a large or medium value of s) and certain materials (with a small slope of the deformability curve), the process of fracture can be modeled by means of the torsion test on a cylindrical specimen with a scratch made along its generating line.
- For modeling the remaining cases of drawing and for other materials the upsetting test can be used.
- Attempts can also be made to model the above cases of drawing by means of the upsetting test, but using different lubricants for particular drawing schedules and particular materials.

- It is possible to model fracture in the process of drawing tubes on a fixed mandrel by taking into account the history of deformation, this being the most correct method from a theoretical point of view. However, this suggestion should be backed by investigation covering different materials. Cases have been known where formulas of lesser accuracy from a theoretical point of view are in better agreement with technological practice. In spite of this restriction, the possibility of modeling the drawing process by means of a mechanical test, instead of by a test with a plane deformation state, as applied hitherto, seems to be very interesting (Ref 10).

6. Summary

A generalized model, as devised by W.L. Kolmogorow (Ref 1, 2), describing the deformability of metal in the process of drawing tubes on a fixed mandrel has been presented. It has been shown that according to this model the coefficient of utilization of reserve of plasticity decreases with the increase of the coefficient of friction (at a constant angle of the die reduction zone). This is a logical error involved in the presented model.

The following proposals relating to further developments of the model have been put forward:

- Elimination of logical error
- Further development of a theory aimed at diminishing assumptions and increasing accuracy in the determination of stresses
- The need to devise a generalized model for numerous types of drawing dies. In recent years there was very rapid development of both common and pressure-type dies (Ref 19-20). The digital computer makes it possible to devise such a generalized model due to the possibility of zeroing certain coefficients and nonperformance of certain procedures.

It has been demonstrated that the coefficient of elongation is not a sufficiently precise measure of deformation in the drawing process.

Table 3 Effect of coefficients from the equation defining the curve of deformability on the coefficient of utilization of plasticity reserve

Item	Dimensions of ingoing tube, mm	Dimensions of finished tube, mm	$e_0 = 1.4306$, $e_1 = e_2 = 0$			$e_0 = 1.071$, $e_1 = -0.5940$, $e_2 = 0.1176$			$e_0 = 0.7153$, $e_1 = -0.5940$, $e_2 = 0.1176$		
			ψ_{eff}	ψ_{calc}	Error	ψ_{eff}	ψ_{calc}	Error	ψ_{eff}	ψ_{calc}	Error
1	57 × 4.2	53 × 2.7	0.444	0.811	82.7	0.444	0.539	21.4	0.444	0.728	64.0
2	57 × 4.2	53 × 3.9	0.205	0.253	23.4	0.205	0.270	32.4	0.205	0.421	105
3	57 × 4.2	36.6 × 2.7	1.193	1.401	17.4	1.193	1.549	29.8	1.193	2.280	90.9
4	57 × 4.2	36.6 × 3.9	0.839	0.828	1.3	0.839	1.131	34.8	0.839	1.765	62.9
5	57 × 4.2	45.6 × 3.36	0.608	0.741	21.9	0.608	0.795	30.8	0.608	1.200	97.4
6	57 × 4.2	45 × 3.4	0.624	0.744	19.2	0.624	0.818	31.1	0.624	1.239	98.8
7	57 × 1.2	45.6 × 0.96	0.597	0.721	20.7	0.597	0.784	31.4	0.597	1.200	109
8	57 × 1.2	53 × 1.12	0.197	0.238	20.8	0.197	0.262	33.0	0.197	0.415	110.4
9	57 × 1.2	36.6 × 1.12	0.839	0.818	2.5	0.839	1.138	35.6	0.839	1.796	114
10	55 × 3.66	48.4 × 2.83	0.448	0.630	40.6	0.448	0.572	27.7	0.448	0.841	87.8
11	55.6 × 3.92	48.4 × 2.83	0.514	0.754	46.8	0.514	0.651	26.6	0.514	0.944	83.8
12	56 × 4.21	48.4 × 2.83	0.571	0.877	53.5	0.571	0.718	25.7	0.571	1.026	79.5
13	56.7 × 4.51	48.4 × 2.83	0.634	1.006	58.6	0.634	0.793	25.1	0.634	1.125	77.6
14	57.2 × 4.82	48.4 × 2.83	0.686	1.124	63.8	0.686	0.854	24.4	0.686	1.201	75.0
15	58.2 × 5.3	48.3 × 2.83	0.770	1.306	69.6	0.770	0.956	24.1	0.770	1.334	73.1

Item	$e_0 = 1.9$, $e_1 = 0$, $e_2 = 0$			$e_0 = 1.425$, $e_1 = -1.1$, $e_2 = 0$			$e_0 = 0.95$, $e_1 = -1.1$, $e_2 = 0$			$e_0 = 2$, $e_1 = 0$, $e_2 = 0$			$e_0 = 1.5$, $e_1 = -0.3$, $e_2 = 0$		
	ψ_{eff}	ψ_{calc}	Error	ψ_{eff}	ψ_{calc}	Error	ψ_{eff}	ψ_{calc}	Error	ψ_{eff}	ψ_{calc}	Error	ψ_{eff}	ψ_{calc}	Error
1	0.346	0.599	73.1	0.346	0.432	24.8	0.346	0.658	90.1	0.459	0.569	25.0	0.459	0.578	26.0
2	0.163	0.188	15.3	0.163	0.227	39.2	0.163	0.423	159.5	0.165	0.178	7.8	0.165	0.215	30.2
3	0.901	1.042	6.7	0.901	1.193	32.4	0.901	1.871	92.1	0.932	0.989	6.1	0.932	1.223	31.2
4	0.649	0.619	4.8	0.649	0.895	37.9	0.649	1.508	132.2	0.591	0.588	0.5	0.591	0.790	33.7
5	0.472	0.551	16.7	0.472	0.637	34.9	0.472	1.075	127.5	0.485	0.523	7.8	0.485	0.635	30.9
6	0.484	0.553	14.2	0.484	0.655	35.4	0.484	1.106	128.8	0.491	0.526	7.1	0.491	0.645	31.4
7	0.467	0.536	14.7	0.467	0.638	36.6	0.467	1.113	138.1	0.473	0.509	7.6	0.473	0.620	31.1
8	0.158	0.177	12.0	0.158	0.223	41.1	0.158	0.432	173.1	0.156	0.168	7.7	0.156	0.205	31.4
9	0.654	0.612	6.4	0.654	0.912	39.4	0.654	1.579	141.2	0.587	0.581	1.0	0.587	0.786	33.9
10	0.349	0.466	33.5	0.349	0.463	32.7	0.349	0.782	124.0	0.390	0.443	13.6	0.390	0.504	29.2
11	0.400	0.558	39.5	0.400	0.525	31.2	0.400	0.863	115.7	0.460	0.531	15.4	0.460	0.592	28.7
12	0.444	0.649	46.2	0.444	0.576	29.8	0.444	0.927	108.9	0.526	0.617	17.3	0.526	0.674	28.1
13	0.493	0.744	50.9	0.493	0.636	29.0	0.493	1.006	104.0	0.596	0.707	18.6	0.596	0.760	27.5
14	0.534	0.832	55.9	0.534	0.684	28.1	0.534	1.066	99.6	0.658	0.790	20.0	0.658	0.837	27.2
15	0.601	0.966	60.7	0.601	0.765	27.2	0.601	1.170	94.6	0.753	0.918	21.9	0.753	0.955	26.8

Note: The error was calculated from the formula $[(\psi_{eff} - \psi_{calc})/\psi_{eff}] \times 100\%$ and ψ_{calc} was calculated for $e_0 \cdot e_1 \cdot e_2 \neq 0$. Source: Ref 6

This paper has presented the possibility of modeling the boundary (critical) deformability in the process of drawing tubes on a fixed mandrel by means of a simple mechanical test.

The conclusions in this paper were drawn by analyzing relations defined by a system of equations, introduced to describe definite characteristics of the process of drawing tubes on a fixed mandrel. It will be interesting to see whether these conclusions are above the model or whether they will change upon further generalization of the model.

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